# Unit Plan – Backward Design (UbD)
## Factoring Polynomials – Chapter 9 - Algebra I

### DESIGNER

<table>
<thead>
<tr>
<th>Name</th>
<th>School</th>
<th>Email</th>
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<tbody>
<tr>
<td>Jeannine Lanphear</td>
<td>Cranbury School</td>
<td><a href="mailto:jlanphear@cranburyschool.org">jlanphear@cranburyschool.org</a></td>
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</tbody>
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### BASIC INFORMATION

<table>
<thead>
<tr>
<th>Unit Title:</th>
<th>Chapter 9 – Algebra I</th>
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<tbody>
<tr>
<td>Unit Theme:</td>
<td>Factoring Polynomial Expressions</td>
</tr>
<tr>
<td>Subject Areas Addressed:</td>
<td>Mathematics - Algebra I</td>
</tr>
<tr>
<td>Content Area Extensions:</td>
<td></td>
</tr>
<tr>
<td>Art</td>
<td>Piet Mondrian’s color block art, contrast non-representational intent of his canvases with algebra tiles which are representational</td>
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<tr>
<td>Technology</td>
<td>uses of technology, satellite dish parabolic curve</td>
</tr>
<tr>
<td>Design</td>
<td>maximizing space in two-dimensional designs, finding dimensions given certain restrictions</td>
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<tr>
<td>Engineering</td>
<td></td>
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<tr>
<td>Architecture</td>
<td>ex: bridge span arches or approximating St. Louis Gateway (catenary arch)</td>
</tr>
<tr>
<td>Packaging Engineering</td>
<td>determining dimensions to meet volume requirements</td>
</tr>
<tr>
<td>Materials Engineering</td>
<td>breaking strength of rope- determining size of rope to meet breaking strength requirements</td>
</tr>
<tr>
<td>Aerodynamics</td>
<td>determining pressure difference required to generate lift</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>determining board feet a log will produce</td>
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<tr>
<td>Civil engineering</td>
<td>water management</td>
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<tr>
<td>Physics</td>
<td>Falling Objects, how long in air, maximum height reached, what time to reach a given height- ex: fireworks, thrown ball, rockets, roller coasters, divers</td>
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<tr>
<td>Marine Biology</td>
<td>determining time in air, distance of dolphin or penguin jump.</td>
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<tr>
<td>Social Studies</td>
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<tr>
<td>History of Conflict</td>
<td>effect of technology on outcome of conflict - ex: Battleship NJ and guns of 1930s used in 20th century conflicts</td>
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<tr>
<td>Modeling data that represents social trends - predicting future values or time to reach future values</td>
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<tr>
<td>Grade Level:</td>
<td>7 – 8</td>
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<tr>
<td>Time Frame:</td>
<td>3 school weeks</td>
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Unit Summary: (broad overarching goals and specific topic goals.)

| Students will use algebra tiles, along with technology tools, to factor polynomial expressions and solve quadratic trinomials. Use of algebra tiles, in both hands-on and virtual manipulative forms, will provide a visual representation and tactile learning experience to assist learners in connecting concrete experiences and representations, to abstract algebraic symbol manipulations. Students will use Web 2.0 tools to virtually peer-teach and to collaborate as a community of learners. These activities will provide opportunities to process learning in this unit multiple times and in multiple forms, to increase learner retention and achievement, and to provide a deeper understanding through critical thinking. |
## I. DESIRED RESULTS

Content Standards Addressed:

<table>
<thead>
<tr>
<th>MATHEMATICS STANDARDS</th>
<th>4.1.8 A. Number Sense</th>
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<tbody>
<tr>
<td>1. Extend understanding of the number system by constructing meanings for the following:</td>
<td></td>
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<tr>
<td>• Rational numbers</td>
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<tr>
<td>• Exponents</td>
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<tr>
<td>• Roots</td>
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<tr>
<td>5. Use whole numbers, fractions, decimals, and percents to represent equivalent forms of the same number</td>
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<thead>
<tr>
<th>4.1.8 B. Numerical Operations</th>
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<tbody>
<tr>
<td>2. Use exponentiation to find whole number powers of numbers.</td>
</tr>
<tr>
<td>3. Find square and cube roots of numbers and understand the inverse nature of powers and roots.</td>
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<tr>
<td>5. Understand and apply the standard algebraic order of operations, including appropriate use of parentheses.</td>
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<thead>
<tr>
<th>4.1.12 B. Numerical Operations</th>
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<tbody>
<tr>
<td>1. Extend understanding and use of operations to real numbers and algebraic procedures.</td>
</tr>
<tr>
<td>2. Develop, apply, and explain methods for solving problems involving rational and negative exponents</td>
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<tr>
<th>4.2.8 A. Geometric Properties</th>
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<tbody>
<tr>
<td>2. Understand and apply the Pythagorean theorem.</td>
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<tr>
<td>3. Understand and apply properties of polygons.</td>
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<tr>
<td>• Quadrilaterals, including squares, rectangles, parallelograms, trapezoids, rhombi</td>
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<tr>
<td>• Regular polygons</td>
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<tr>
<td>5. Use logic and reasoning to make and support conjectures about geometric objects.</td>
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<thead>
<tr>
<th>4.2.12 A. Geometric Properties</th>
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<tbody>
<tr>
<td>1. Use geometric models to represent real-world situations and objects and to solve problems using those models (e.g., use Pythagorean Theorem to decide whether an object can fit through a doorway).</td>
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<tr>
<td>3. Apply the properties of geometric shapes.</td>
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<thead>
<tr>
<th>4.2.12 B. Transforming Shapes</th>
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<tr>
<td>1. Determine, describe, and draw the effect of a transformation, or a sequence of transformations, on a geometric or algebraic representation, and, conversely, determine whether and how one representation can be transformed to another by a transformation or a sequence of transformations.</td>
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<tr>
<th>4.2.12 E. Measuring Geometric Objects</th>
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<tbody>
<tr>
<td>1. Use techniques of indirect measurement to represent and solve problems.</td>
</tr>
<tr>
<td>2. Use a variety of strategies to determine perimeter and area of plane figures and surface area and volume of 3D figures.</td>
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<thead>
<tr>
<th>4.3.8 A. Patterns</th>
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<tbody>
<tr>
<td>1. Recognize, describe, extend, and create patterns involving whole numbers, rational numbers, and integers.</td>
</tr>
<tr>
<td>• Descriptions using tables, verbal and symbolic rules, graphs, simple equations or expressions</td>
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<thead>
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<th>4.3.12 A. Patterns</th>
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<tr>
<td>3. Use inductive reasoning to form generalizations.</td>
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<tr>
<th>4.3.8 C. Modeling</th>
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<tbody>
<tr>
<td>2. Use patterns, relations, symbolic algebra, and linear functions to model situations.</td>
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<tr>
<td>• Using concrete materials (manipulatives), tables, graphs, verbal rules, algebraic expressions/equations/inequalities</td>
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<th>4.3.12 C. Modeling</th>
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<tr>
<td>1. Use functions to model real-world phenomena and solve problems that involve varying quantities.</td>
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• Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of mailing a first-class letter over the past 200 years)
• Expressions, equations and inequalities
• Same function can model variety of phenomena
• Applications in mathematics, biology, and economics (including compound interest)

4.3.8 D. Procedures
2. Solve simple linear equations informally, graphically, and using formal algebraic methods.
   • Multi-step, integer coefficients only (although answers may not be integers)
   • Using paper-and-pencil, calculators, graphing calculators, spreadsheets, and other technology

4. Create, evaluate, and simplify algebraic expressions involving variables.
   • Order of operations, including appropriate use of parentheses
   • Distributive property
   • Substitution of a number for a variable
   • Translation of a verbal phrase or sentence into an algebraic expression, equation, or inequality, and vice versa

5. Understand and apply the properties of operations, numbers, equations, and inequalities.
   • Additive inverse
   • Multiplicative inverse
   • Addition and multiplication properties of equality
   • Addition and multiplication properties of inequalities

4.3.12 D. Procedures
1. Evaluate and simplify expressions.
   • Add and subtract polynomials
   • Multiply a polynomial by a monomial or binomial
   • Divide a polynomial by a monomial
   • Perform simple operations with rational expressions
   • Evaluate polynomial and rational expressions

2. Select and use appropriate methods to solve equations and inequalities.
   • Linear equations and inequalities – algebraically
   • Quadratic equations – factoring (including trinomials when the coefficient of \( x^2 \) is 1) and using the quadratic formula

4.5 A. Problem Solving
1. Learn mathematics through problem solving, inquiry, and discovery.
2. Solve problems that arise in mathematics and in other contexts.
   • Open-ended problems
   • Non-routine problems
   • Problems with multiple solutions
   • Problems that can be solved in several ways

3. Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.

4. Pose problems of various types and levels of difficulty.

5. Monitor their progress and reflect on the process of their problem solving activity.

4.5 B. Communication
1. Use communication to organize and clarify their mathematical thinking.
   • Reading and writing
   • Discussion, listening, and questioning

2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.

3. Analyze and evaluate the mathematical thinking and strategies of others.

4. Use the language of mathematics to express mathematical ideas precisely.

4.5 C. Connections
1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4. Apply mathematics in practical situations and in other disciplines.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

**4.5 D. Reasoning**
2. Use reasoning to support their mathematical conclusions and problem solutions.
6. Evaluate examples of mathematical reasoning and determine whether they are valid.

**4.5 E. Representations**
1. Create and use representations to organize, record, and communicate mathematical ideas.
   - Concrete representations (e.g., base-ten blocks or algebra tiles)
   - Pictorial representations (e.g., diagrams, charts, or tables)
   - Symbolic representations (e.g., a formula)
   - Graphical representations (e.g., a line graph)
2. Select, apply, and translate among mathematical representations to solve problems.
3. Use representations to model and interpret physical, social, and mathematical phenomena.

**4.5 F. Technology**
1. Use technology to gather, analyze, and communicate mathematical information.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
5. Use computer software to make and verify conjectures about geometric objects.

**TECHNOLOGICAL LITERACY STANDARDS**

**8.1.8 A. Basic Computer Skills and Tools**
1. Use appropriate technology vocabulary.
2. Use common features of an operating system (e.g., creating and organizing files and folders).
3. Demonstrate effective input of text and data, using touch keyboarding with proper technique.
4. Input and access data and text efficiently and accurately through proficient use of other input devices, such as the mouse.
5. Create documents with advanced text-formatting and graphics using word processing.
8. Design and produce a basic multimedia project.
10. Use network resources for storing and retrieving data.

**8.1.8 B. Application of Productivity Tools**

**Social Aspects**
2. Exhibit legal and ethical behaviors when using information and technology, and discuss consequences of misuse.
3. Explain the purpose of an Acceptable Use Policy and the consequences of inappropriate use of technology.
4. Describe and practice safe Internet usage.
5. Describe and practice “etiquette” when using the Internet and electronic mail.

**Information Access and Research**
6. Choose appropriate tools and information resources to support research and solve real world problems, including but not limited to:
   - On-line resources and databases
   - Search engines and subject directories
7. Evaluate the accuracy, relevance, and appropriateness of print and non-print electronic information sources.

**Problem Solving and Decision Making**
8. Use computer applications to modify information independently and/or collaboratively to solve
Overarching Big Ideas/Enduring Understandings:

Students will understand that…
- multiplying polynomials and factoring polynomials are inverse operations, recognizing that factoring is a form of division
- second degree expressions and equations model real-life applications.
- geometry can be used to model algebraic expressions using area models
- patterns observed when multiplying polynomials are used in reversing the multiplication process when factoring
- mathematical ideas interconnect and build on one another to produce a coherent whole
- completely factoring a polynomial expression is akin to prime factorization of a composite number
- algebraic methods build by scaffolding skills and more difficult problems are solved by manipulating them into a simpler problem that can more easily be solved with prior knowledge; for example, the grouping method is applied twice when an expression is re-written using the x-box method of factoring; two linear equations are generated to be solved when solving a quadratic equation by factoring and applying the Zero Product Property
- variable solutions from equations modeling a problem must be interpreted; negative solutions or expressions that evaluate to negative values can’t represent dimensions but number theory problems allow negative solutions.

Conceptual Essential Questions:
(Questions to guide the unit and focus the teaching and learning.)
- How does an understanding of factors as multiplication building blocks transfer from constants to polynomial algebraic expressions?
- How are the solutions to an equation related to the solutions to a problem?
- How are the patterns observed when multiplying polynomials used in reversing the multiplication process when factoring?
- How do algebra tiles model abstract procedures for 5 factoring methods?
- What does it mean to be a perfect square and are only integers perfect squares?
- How are multiplying, dividing and factoring related?
- How do you know when an expression is completely factored?
- How do you choose an appropriate and efficient factoring method for a given polynomial?
- How does the number of terms in a polynomial relate to selecting a possible factoring method?
- How can the form of a problem be changed into an equivalent yet simpler problem?
- How can a community of learners collaborate to create a body of learning about a specific topic, and navigate and prune the overwhelming resources available on the Internet?

Knowledge & Skills:
(What key knowledge and skills will students acquire as a result of this unit?)

Students will know…
- that some expressions require the application of more than one form of factoring to be completely factored
- that fractions can be perfect square expressions
- that squaring a binomial always produces a trinomial
- that before applying the Zero Product Property, all terms must be on one side of the equation with zero isolated on the other side.
- that some polynomial expressions are prime, even though they appear to be composite by addition
- how to model and solve a vertical motion problem using quadratic equations and the zero product property.
## Prior Knowledge and Skills Needed to Achieve Desired Outcomes

- Prime and Composite Numbers
- Factors, Factor Pair Listing Method and Prime Factorization
- Greatest Common Factor (GCF)
- Properties and Patterns of Integer Multiplication
- Multiplying Polynomials and patterns observed when multiplying binomials (FOIL, square of a sum, square of a difference, product of a sum and a difference)
- Division of Monomials
- Solving Linear Equations
- Finding Degree of polynomials
- Using variables and writing algebraic expressions to model word problems
- Basic algebra tiles representations
II. Evidence of Expected Outcomes

Culminating Performance Task
What evidence will show that students understand?

Project-based learning:
• Internet Resources posting / Feedback on another student’s posting as assigned
• Factoring Method Tutorial / Feedback on another student’s posting as assigned
• Frayer Model Vocabulary Term posting / Feedback on another student’s posting as assigned

Performance Task Rubric
Rubrics are available as attachment for:
• Algebra tiles virtual manipulatives tutorial and peer feedback
• Vocabulary term Frayer Model posting and peer feedback
• Internet Resources posting and peer feedback
• Entire unit rubric

Other Assessment Evidence
(Quizzes, Tests, Prompts, and/or Work Samples)
• Senteo entrance tickets/quizzes
• Mid-Chapter test
• Graded homework/work samples
• Chapter test
• Word Problems take-home activity
• Foldable study activity
• Teacher observation
• Meta-cognitive study activity

III. Learning Experiences
What sequence of teaching and learning experiences will equip students to develop and demonstrate the desired understandings?

Unit Hook/Preview Activity:
Algebra tiles “play time”, followed by introduction to algebra tiles movie.

Sequence of Learning Activities
List the learning activities in the sequence that they will be addressed.

Time Line is available as an attachment.
Dashes separate possible learning periods.
Students complete an introduction to the algebra tiles through hands-on tiles design time and a SMART Board activity.
Discussion on uses of factoring to provide sense and meaning; Zero Product Property once a polynomial is factored can be used to solve equations.
Project and milestones are explained to students; rubrics and timelines distributed
Students review requisite knowledge on prime factorization and factoring and transfer to algebraic examples.
Students practice with tile.
Instructions to students for foldable.

--
Students complete algebra tiles challenge based on method of the day - GCF/Grouping method.
Students participate in SMART Board lesson on GCF/Grouping method.
Students practice GCF/grouping method.
Students receive instruction on use of web-site to download templates.
Students download templates onto jump drives.
---
Students take Senteo entrance ticket on factoring using GCF/Grouping method- concrete and abstract representations.
Review Senteo answers.
Students split into two groups based on results:
a) success group- continue to practice with abstract examples; early finishers work on projects
b) re-teach group- continue to practice with tiles with teacher assistance on transfer to abstract.
---
Students complete algebra tiles challenge based on method of the day- \( x^2 + bx + c \) method.
Students participate in SMART Board lesson on patterns in \( x^2 + bx + c \) method.
Students practice method of the day.
---
Students take Senteo entrance ticket on factoring using \( x^2 + bx + c \) method- concrete and abstract representations.
Review Senteo answers.
Students split into two groups based on results:
a) success group- continue to practice with abstract examples; early finishers work on projects
b) re-teach group- continue to practice with tiles with teacher assistance on transfer to abstract.
---
Students complete algebra tiles challenge based on method of the day - \( ax^2 + bx + c \) (x-box).
Students participate in SMART Board lesson on method.
Students practice method of the day.
---
Students take Senteo entrance ticket on factoring using \( ax^2 + bx + c \) method- concrete and abstract representations.
Review Senteo answers.
Students split into two groups based on results:
a) success group- continue to practice with abstract examples; early finishers work on projects
b) re-teach group- continue to practice with tiles with teacher assistance on transfer to abstract.
---
Students complete algebra tiles challenge based on method of the day – differences of squares.
Students participate in SMART Board lesson on method.
Students practice method of the day.
---
Students take Senteo entrance ticket on factoring using differences of squares- concrete and abstract representations.
Review Senteo answers.
Students split into two groups based on results:
a) success group- continue to practice with abstract examples; early finishers work on projects
b) re-teach group- continue to practice with tiles with teacher assistance on transfer to abstract.
---
Students complete algebra tiles challenge based on method of the day – perfect square trinomials.
Students participate in SMART Board lesson on method.
Students practice method of the day.
---
Students take Senteo entrance ticket on factoring using perfect square trinomials- concrete and abstract representations.
Review Senteo answers.
Students split into two groups based on results:
a) success group - continue to practice with abstract examples; early finishers work on projects
b) re-teach group - continue to practice with tiles with teacher assistance on transfer to abstract.

Challenge on perfect square trinomials – find the missing term
Mini-lesson on finding missing term when: a) middle term is missing  b) last term is missing
Project work time
Provide students with instruction on posting feedback to projects.
Provide students with instructions on posting projects.

Mid-chapter test- mixed representations, problem states method to use.
Project work time

Students complete word problems challenge to try to write equations for situations.
Lesson on solving word problems and interpreting answers to quadratic equations that model real-life situations.
Review mid-chapter test results.
Students practice word problems.

Students complete fractional perfect squares challenge.
Students complete group work activity on determining the best method.
Discuss need to always check GCF first, factor completely.
Students present group work and complete organizer on choosing method.
Practice mixed methods and using more than one type of factoring.

Student work time/ Q&A

Chapter test – including essay on using geometric models
Word problems take-home activity

**Materials/Resources Needed:** (related websites, reference materials, worksheets, etc.)

- Movie on Algebra tiles
- SMART Board file/lesson for each section (1-6) with accompanying handout
- Set of algebra tiles for each student
- SMART Board Notebook template file for algebra tiles
- Frayer model for vocabulary word document template file
- Cranbury Acceptable Use Policy document
- Class Guidelines for Web 2.0 document
- Computers with SMART Board Notebook software, Microsoft Word, and Internet browser installed
- Student usernames with password for school network system
- Student task organizer sheet
- Student timeline and due date sheet
- List for making and recording assignments to students on projects
- Algebra Lab Challenge Puzzles
- Punchline Algebra Book B Section 13 puzzles with combined area model and algebraic representations (self-checking)
- Traditional symbolic Algebra factoring practice sheets from Resource book
- Senteo remotes
- Word Problems Take-Home Practice sheet
- Calculators
- Senteo quizzes and printouts (6 sections)
- Mid-chapter test
- Word Problem practice sheets
- Section challenge sheets for each day
- Bellwork pages/ Entrance tickets for each day
- 8-1/2 inch by 14 inch blank paper for foldable
- Homemade algebra tiles on stock paper for home use
- Group work packets for determining which method to use
- Factoring method organizer for group work day
- Practice multiple choice chapter test
- Traditional chapter test
- Computer-based project rubrics (3)
- Unit rubric
- National Library of Virtual Manipulatives website:
  [http://nlvm.usu.edu/en/nav/category_g_4_t_2.html](http://nlvm.usu.edu/en/nav/category_g_4_t_2.html)

**Textbook and Other References:**

  - [http://www.glencoe.com/sec/math/algebra/algebra1/algebra1_05](http://www.glencoe.com/sec/math/algebra/algebra1/algebra1_05)
- *Punchline Algebra: Book B*, Marcy Mathworks, 2006
- *Algebra Tiles for the Overhead Projector*, Hilde Howden, Cuisenaire, 1994
- *Pre-Algebra and Algebra I using Algebra Models*, Elyce B. Duerr, Classroom Products, 2005